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LIFE, WORK, AND PAPERS OF WOLFGANG EBELING (1951–2025)

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Wolfgang Ebeling, Segovia 2006

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Prof. Dr. Wolfgang Ebeling was born on 05.10.1951. He died (too early) on 06.01.2025. Wolfgang devoted his life to his family, to his students and his teaching, and to his scientific work in mathematics. He was reliable, good-hearted, quiet, honest and conscientious. We remember his community spirit, his gentleness and his mathematical results. He left his wife Bettina (née Zschoche), his children Dr. Bastian Ebeling and Mirja Ebeling and four grandchildren. This text reports on his life and work. At the end it lists his papers.

His life.

Wolfgang was born in Bückeburg in Lower Saxony as the only child of an engineer. During his childhood and school years his family moved several times, to Wilhelmshaven, Kassel, Weilheim and Sankt Augustin (near Siegburg). He passed his Abitur in 1971 in Siegburg. Having served ten years in the THW (technisches Hilfswerk), he did not have to do military service or civil service after the Abitur.

After the Abitur Wolfgang moved to Bonn, where he stayed until spring 1987, with an interruption of one year in Utrecht 1982–1983 and two months in Chapel Hill in spring 1986.

He studied mathematics, passing the Diplomhauptprüfung in Bonn in 1977. Wolfgang wrote both his diploma thesis [E77] and his doctoral thesis [E80] under the guidance of Egbert Brieskorn. The subject of his diploma thesis, Dynkin diagrams of isolated hypersurface singularities, was to accompany him throughout his entire scientific life.

After finishing his doctoral thesis in 1980, he was scientific assistant of Brieskorn at the University of Bonn 1980–1982 and 1983–1985. Wolfgang spent the academic year 1982–1983 as a postdoctoral researcher at the University of Utrecht. 1985–1987 he was back in Bonn, interrupted by a two months stay in Chapel Hill, funded by a research grant of the DFG (Deutsche Forschungsgemeinschaft ∼ German Research Council).

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In 1986 Wolfgang completed his habilitation thesis [E87-2]. His first big scientific project, about which we report below, was started in his doctoral thesis [E80] and finished in his habilitation thesis [E87-2].

In 1987–1988 Wolfgang made an excursion into industry. He spent one and half a year as a systems analyst at the Ford Motor Company in Cologne. His time in industry had a lasting impact on him as a scholar. It broadened his view on life significantly and contributed to making him the wonderful, far-sighted, calm and understanding mentor for all of his students – for the stronger and less strong ones, many of whom are now working in industry. He also drew on this experience when teaching calculus classes for engineering students in Hannover, where he was highly respected by the students as a strict, but fatherly teacher with clear standards and a dry sense of humor (at the expense of mathematicians, not engineers).

1988–1990 he was associate professor (UHD) at Eindhoven University of Technology. He always remembered his time in the Netherlands, in Utrecht and in Eindhoven, as a good time for him personally and mathematically. He stayed in touch with a number of his colleagues from that time. At conferences one could often hear him speak Dutch. Indeed, Wolfgang never lost his knowledge of the Dutch language and liked to practice it, whenever the opportunity arose.

In the autumn of 1990 Wolfgang Ebeling became professor at Leibniz University Hannover. He kept this position for 29 1/2 years until his retirement in spring 2020. It fitted well that half a year earlier, Klaus Hulek had also moved to Hannover. Together, they established a research group in algebraic and complex geometry.

During his time in Hannover Wolfgang was involved in a number of research projects. The first major project was the DFG priority programme *Komplexe Mannigfaltigkeiten*, which Ebeling and Hulek had each brought with them to Hannover. This was also the time when boarders opened up in Europe and Wolfgang could use funds from this DFG project to embark on his long lasting and extremely fruitful cooperation with Sabir Gusein-Zade. Starting from 1992 Sabir became a regular visitor in Hannover.

The DFG priority programme was succeeded by two major European projects, namely AGE (Algebraic Geometry in Europe) 1994 – 1998 funded in the EU program Human Capital and Mobility (HCM) and, following this, EAGER (European Algebraic Geometry Research Network) 2000-2004 within the EU's 5th Framework Programme. Wolfgang was involved in all of these projects. The latter EU project still lives on in the form of the EAGER mailing list which is used by practically all European algebraic geometers (and beyond). Later, both Ebeling and Hulek played a major role in a further DFG priority programme, namely *Global Methods in Complex Geometry* SPP 1094 which run from 2000 – 2008 and whose overall coordinator was Thomas Peternell from Bayreuth.

Wolfgang was also crucially involved in the DFG funded Research Training Group (Graduiertenkolleg) *Analysis, Geometry and String Theory* (GRK 1463) in Hannover in the years 2008 – 2017, which brought together researchers from Analysis, Differential Geometry and String Theory. This was a very successful project and several of Wolfgang's PhD students were funded by this project. Altogether, Wolfgang had seven doctoral students in Hannover, all of whom received his undivided attention and full support.

Further research projects included the INTAS project on *Singularities, Bifurcations and Monodromy* from 2001 – 2003, and the DFG priority programme *Representation Theory* (SPP 1388) from 2009 – 2015. Besides these coordinated programmes Wolfgang acquired a number of individual DFG grants. These funded, in particular, his ongoing cooperation with Sabir Gusein-Sade and from 2010 onwards also Atsushi Takahashi. Wolfgang and his coauthors were very efficient in their cooperation. Typically, Sabir or Atsushi would come for some time to Hannover. During such a visit they would discuss and collaborate on an almost daily basis, and, very reliably, soon after a new preprint would appear on arXiv. Gusein-Sade and Takahashi were the two mathematicians with whom Wolfgang Ebeling cooperated most closely, but he also had other coauthors – we will discuss this in more detail in the section about his work.

In 2007 the North German Algebraic Geometry seminar was founded. This started out as a meeting between Göttingen, Hannover and the International University of Bremen (the BGH seminar), but soon became a meeting including most universities in the north of Germany and beyond, even including Groningen in the Netherlands. It was Wolfgang who invented the acronym NoGAGS.

Wolfgang was a committed academic teacher. He gave very clear lectures putting a lot of effort in preparation. He was very kind to his students and much beloved by them. Some of his students kept up contact with him after they left the university.

Although his heart was in mathematics, Wolfgang did not shirk his responsibilities as an administrator. Most notably, he was dean of the Fachbereich of Mathematics from 2000 – 2002. This was a time of major reorganization, as the computing department left mathematics to establish, together with researchers from electrical engineering, a new department of computer science. It was Wolfgang's duty to handle the necessary negotiations, a task which he achieved admirably. Besides, Wolfgang served on numerous committees and the examination board. Wolfgang was unanimously esteemed by his colleagues – he was always very reliable and very considerate and kind to others.

Wolfgang and his family settled in Burgdorf, a small market town near Hannover. There, they were very well integrated and had a rich social life. Watching football at the Ebeling house, in particular during a European or a World cup became a very popular event. Wolfgang also became a keen member of the Burgdorf Kantorei. Singing in this choir gave him lots of joy and he took part in the performances of many great choral works, such as Bach's St John passion or Mozart's requiem.

In later years, Bettina and Wolfgang also provided a home for a young Afghan refugee whom they helped greatly to integrate into German society.

Wolfgang had many plans for his retirement, both mathematically and privately. Unfortunately, many of these could not be realized. His retirement coincided more or less with the beginning of the first of the covid lockdowns. In the last year of his life his illness, which he faced very bravely, increasingly required his full strength.

His work.

Wolfgang Ebeling wrote 83 published scientific papers, three books, a diploma thesis, a doctoral thesis and three papers for the research journal of Leibniz University Hannover. His habilitation thesis later became his first book. Besides, he was coeditor of one proceedings volume of a conference in Hannover.

At several stages, Wolfgng wrote surveys about his work and related work. Here we mention eight surveys, his first book [E87-2], his third book [E01] and the six articles [E06-1], [EG06-3], [EG07-2],[E17], [E20] and [EG23].

Wolfgang Ebeling's research centred around three big scientific projects:

- The monodromy groups of isolated singularities of hypersurfaces and of complete intersections of even dimension.
- Indices of holomorphic vector fields and 1-forms on singular varieties.
- Duality statements for special quasihomogeneous singularities, which fit into mirror symmetry for singularities.

Not all, but many of his papers fit into these projects. The second and third project accompanied him throughout his entire life.

Wolfgang pursued the second project together with Sabir Gusein-Zade, with whom he had a long-lasting cooperation. 43 out of Wolfgang Ebeling's 83 published papers were written together with him.

He conducted the third project together with Sabir Gusein-Zade and Atsushi Takahashi. With Atsushi Takahashi he wrote 9 joint papers (one with Gusein-Zade and Takahashi). Below, we will give more details about the second and third project.

Wolfgang Ebeling solved the first big project completely. He started it in his doctoral thesis [E80] and finished it in his habilitation thesis [E87-2].

The Milnor lattice of an isolated hypersurface singularity of even dimension comes equipped with an even intersection form. The monodromy group is a subgroup of the automorphism group of the Milnor lattice with this intersection form. In his doctoral thesis [E80] Ebeling showed, for large families of singularities, that and how the monodromy group is determined by the Milnor lattice with intersection form alone. This generalized an observation of Pinkham for the 14 exceptional singularities. His proof used an algebraic result of Kneser [Kn81] and relied on a good control of the Dynkin diagrams of these families of singularities. The papers [E81] (an abridged version of [E80]), [E83-1] and [E83-2] document this.

Ideas of Looijenga and Janssen [Ja83] for the case of singularities of odd dimension allowed Ebeling to give a stronger algebraic result, and to extend his work to all isolated hypersurface singularities except the hyperbolic ones (where the result does not hold) and to intersections of two quadrics in $\mathbb{C}^4$ [E84]. Finally, in his habilitation thesis [E87-2], of which [E87-1] is an abridged version, he was able to extend the result to all isolated complete intersection singularities, except the hyperbolic ones (where it does not hold).

[E87-2] is the first of three books which he wrote. We will now first say something about the other two books. Afterwards, in the report on his other published papers, we will not go on strictly chronologically, but order the papers by projects, first treating the smaller and then the larger projects.

His second book [E94] on *Lattices and codes* became extremely popular. It is based on a course with the same title which he gave in his first semester in Eindhoven, the winter semester 1988/89. Several of his colleagues in Eindhoven were specialists for coding theory. The book further builds on a course of Hirzebruch on *Coding theory and relations with geometry* (the course had been given in German two years before at the University of Bonn) and Wolfgang Ebeling's own inaugural lecture on *The Leech lattice* in January 1987 at the University of Bonn.

His third book [E01] was translated into English [E07]. It builds on courses which he held in the 1990's at the University of Hannover, several courses on function theory and one course on singularity theory. The book contains chapters on Riemann surfaces, holomorphic functions in several variables, differential topology and isolated hypersurface singularities.

Now we report on four rather isolated papers.

A Coxeter-Dynkin diagram of an isolated hypersurface singularity encodes the intersection matrix of a distinguished basis of the Milnor lattice. From this one can read off the Seifert form, the intersection form and the monodromy. Still, one can hope to extract also more transcendent information from it, like the modularity or deformation relations to smaller singularities. Il'yuta [Il87] had formulated some interesting, but too optimistic conjectures on this, which were based on Il'yuta's *monotone cycles* in a Coxeter-Dynkin diagram. Ebeling [E96] confirmed them for the unimodal and simple singularities, but gave a bimodal counterexample.

Before [E18-1], it was well known that a simple singularity has only finitely many Coxeter-Dynkin diagrams and only finitely many distinguished bases, and that a simple elliptic singularity has finitely many Coxeter-Dynkin diagrams, but infinitely many distinguished bases. Motivated by a question of Hertling, [E18-1] proves that each other singularity has infinitely many Coxeter-Dynkin diagrams and thus infinitely many distinguished bases. More precisely, for each other singularity each integer appears in the intersection matrix of a suitable distinguished basis.

Wolfgang Ebeling and Joseph Steenbrink [ES98] constructed spectral pairs for each isolated complete intersection singularity. Compared to Steenbrink's construction of spectral pairs for isolated hypersurface singularities [St76] more than 20 years earlier, two difficulties arose, for which [E87-2] suggested solutions. One has to work with a certain *relative* cohomology group, and one has to use a certain *smoothing pair* with some extremal property, which is not completely canonical, but good enough.

In [BBE09] Ragnar Buchweitz, Wolfgang Ebeling and Hans Christian Graf von Bothmer offer a panorama of results on free divisors in base spaces of families of singularities. They concentrate on the cases where the fiber dimension is zero, one or two.

The paper [E90] was the prelude to a cooperation of Wolfgang Ebeling with Christian Okonek, which resulted in three joint papers on complex surfaces [EO90] [EO91] [EO94]. In the first three papers [E90] [EO90] and [EO91] Ebeling's expertise on monodromy groups of isolated complete intersection singularities was useful.

In [E90] he presented two complete intersection surfaces which are homeomorphic, but not diffeomorphic. In order to apply results of Friedman and Donaldson, one needs that the monodromy groups of these surfaces are *big*, which follows from [E87-2].

[EO90] and [EO91] use similar arguments in order to study the image of $O(H_2(X,\mathbb{Z}),q)$ in the *component group* $\mathrm{Diff}_+(X)$ for certain algebraic surfaces X with $p_g > 0$.

Though [EO94] does not use Ebeling's expertise on monodromy groups of isolated complete intersection singularities. It studies complex surfaces whose underlying topological manifolds are $S^1 \times \Sigma^3$ with Σ^3 an (integral or rational) homology sphere.

The collaboration of Wolfgang Ebeling and Sabir Gusein-Zade started in 1992. At that moment both were very interested in Dynkin diagrams and monodromy groups of singularities, including non-hypersurface ones. The first discussions were motivated by computations from [E87-2], where Dynkin diagrams were computed for all simple isolated complete intersection singularities except Z_9 and Z_{10}. This led to the results published in [EG95], [EG96], and [EG98].

The second big project, *indices of vector fields and of 1-forms and of their collections on (usually singular) varieties*, was pursued jointly by Ebeling and Gusein-Zade. It was started at the end of the 90's and was completed by the survey [EG23]. The earlier survey [EG06-3] documented the state of the art at that time.

The first step of this long-lasting programme (published in [EG99]) was initiated in a paper by C. McCrory and A. Parusiński [MP97]. The main dream of the programme (not completely achieved), was to produce formulas for indices in terms of signatures of some quadratic forms in the spirit of the celebrated results of D. Eisenbud and H.I. Levine [EL77] and of G.N. Khimshiashvili [Kh77].

One of the achievements of the project (first explained in [EG01], [EG03-1], and [EG03-2]) was the idea to consider indices of 1-forms instead of vector fields. On smooth manifolds, there is essentially no difference in these approaches, whereas on singular varieties the difference becomes essential. For example, vector fields with isolated singular points may not exist on a singular variety germ, but 1-forms always exist. One more difference is related to the fact that the corresponding constructions are usually assumed to be applied to functions. In the context of vector fields, this means application to vector fields. In the context of 1-forms it means application to differentials of functions. However, on singular varieties, differentials are defined in the standard form, whereas an analogue of a gradient vector field demands an individual construction and is not absolutely well-defined.

In this framework, the radial index and the Euler obstruction [EG05-3] were defined and studied, as were the GSV (Gomez-Mont–Seade–Verjovski) index [EG01] [EG03-1] and the homological index [EGS04], all of a 1-form on a singular variety. Also, in [EG09] some key notions related to indices of vector fields and 1-forms on determinantal singularities were defined. These were later further elaborated on and extended by a number of researchers.

Another achievement was to treat indices, not of individual vector fields or 1-forms, but of collections of them. The usual indices are related with the Euler characteristic of a variety, that is with the top Chern number ($\langle c_n, [M^n]\rangle$) of a complex analytic variety. In a similar way, indices of collections of vector fields or of 1-forms are related with other Chern numbers. The authors defined and studied an analogue of the GSV-index [EG07-3] and the so-called Chern obstruction for such collections [EG07-1].

One more part of this project was devoted to equivariant versions of the notions discussed above. This is treated in the papers [EG15-1], [EG15-2], [EG17-2], [EG18-1], [EG18-4].

One paper, which is part of this project, was not written with Sabir Gusein-Zade, but with Xavier Gomez-Mont and Hans-Christian Graf von Bothmer [BEG08].

Before we come to the third big project, we would like to mention a related fourth project on *Poincaré series and monodromies (and induced polynomials) of singularities*.

A relevant indicator was the result of Gusein-Zade, Delgado and Campillo [CDG99] that the Poincaré series of the ring of functions on an irreducible plane curve singularity coincides with the zeta function of its monodromy.

In [E02] Ebeling was able to write the Poincaré series of a two-dimensional quasihomogeneous hypersurface singularity as a quotient of two polynomials with some meaning of their own, one being related, by a duality of K. Saito, to the monodromy. This result was generalized and reconsidered in a number of papers. Indeed, it was extended to quasihomogeneous hypersurface singularities of arbitrary dimension [EG02], to quasihomogeneous complete intersection singularities [EG04-2], to quasihomogeneous surface singularities which are not necessarily hypersurface singularities (also called *Fuchsian singularities*) [E03], and to the simple and unimodal boundary singularities [E09].

The two joint papers [EP10-1] and [EP10-2] of Wolfgang Ebeling and David Ploog reconsidered and reproved in a more conceptual way the results in [E02] and [E03] for the ADE surface singularities and the Fuchsian singularities. This used ideas of Lenzing [Le99].

In [E02], the result for the ADE surface singularities had been proved using the McKay correspondence. In [E18-2] Ebeling carried out a similar construction for the quotient singularities of $\mathbb{C}^3$ by 14 finite subgroups of $SL_3(\mathbb{C})$.

In the two joint papers [EG05-2] and [EG06-1] with Gusein-Zade, the authors calculated Poincaré series, from an arc filtration, for the singularities in Arnold's lists. In the end, this turned out to be quite a coarse invariant.

In the first of their three joint papers [EG10] [EG11-1] [EG12-4], Ebeling and Gusein-Zade constructed a multi-index filtration and an induced Poincaré series for Newton nondegenerate hypersurface singularities. This is difficult to compute. In the second and third paper they considered, in the case of plane curve singularities, a generalization to arbitrary plane curve singularities and made many calculations.

The third big project had two phases, the phase before meeting Atsushi Takahashi and the phase during cooperation with him.

The first phase built on an observation by Arnold [Ar75] on the 14 quasihomogeneous exceptional unimodal singularities, namely his *strange duality*. Indeed, these 14 singularities are Fuchsian, and more precisely triangle singularities. This leads to a triple of Dolgachev numbers for each of these singularities. Similarly, the existence of some special Coxeter-Dynkin diagram gives rise to a triple of Gabrielov numbers for each of these singularities. Arnold observed a duality within the 14 singularities which exchanges these triples.

In their joint paper [EW85], written ten years after Arnold's work, Wolfgang Ebeling and Terry Wall were able to extend this strange duality to some further Fuchsian singularities which are complete intersections and triangle singularities or quadrangle singularities, and also to some virtual singularities. All of these come equipped with triples or quadruples of Dolgachev numbers and of Gabrielov numbers. Remarkably, these tuples are interchanged by a duality. This extension of Arnold's strange duality was the starting point for Ebeling's interest in this project.

His next steps were motivated by two independent observations on the strange duality for the 14 quasihomogeneous singularities. Kobayashi [Ko08] (the preprint is from 1995) found a duality for the weight systems and exponents of monomials of the corresponding singularities. K. Saito [Sa98] further exhibited a duality between the characteristic polynomials of the corresponding singularities. This uses a recipe of his how to obtain from one product of cyclotomic polynomials another such product.

In [E99-1] Ebeling showed that both enhancements to Arnold's strange duality also hold for the extension of Arnold's strange duality in [EW85]. In [E00] he showed that K. Saito's generalization follows in all considered cases from Kobayashi's enhancement. In [E02] and [E03], which we mentioned already above for the fourth project, he further related the Poincaré series of Fuchsian singularities which are hypersurface singularities or complete intersection singularities with the help of K. Saito's duality to the characteristic polynomials.

In [E06-2] Ebeling considered the 95 quasihomogeneous singularities in four variables which are related to K3 weighted projective hypersurface, and which were discovered by Reid and rediscovered by Yonemura. Using again Kobayashi's duality, he could confirm some known mirror symmetry duality in certain pairs of these K3 hypersurfaces.

Atsushi Takahashi has some background in mathematical physics and mirror symmetry which led to new subprojects. He introduced Wolfgang Ebeling and Sabir Gusein-Zade to the notion of invertible polynomials which are quasihomogeneous polynomials with an isolated singularity, the same number of variables and monomials and an invertibility condition for the matrix of exponents. Berglund and Hübsch [BH93] proposed a duality for invertible polynomials which embraces Kobayashi's duality. Berglund and Henningson [BH95] enhanced this to a duality between pairs (f, G) and (f^T, G^T) where f and f^T are dual invertible polynomials and G and G^T are dual finite groups of diagonal and linear coordinate changes. These BHH-dualities are considered to be a mirror symmetry for singularities. In papers by Berglund, Hübsch and Henningson only "physically motivated" cases were considered. E. g., it was assumed that the symmetry groups G and G^T were contained in $SL_n(\mathbb{Z})$. In a series of papers [EG12-3] [EG17-1] [EGT16] it was shown that some mirror-like symmetries always hold, without the assumption that G and G^T are contained in $SL_n(\mathbb{Z})$.

Later, Takahashi proposed an extension from the groups G and G^T to non-abelian groups, namely semi-products of these groups with subgroups of the group of permutations of variables. However, the construction did not work in general. A condition which seems to be necessary and sufficient for a mirror-like duality in this case (a special condition on the subgroup of a permutation group called PC: parity condition) was found in [EG21].

This setting was the starting point for 9 joint papers by Ebeling and Takahashi and 13 joint papers by Ebeling and Gusein-Zade on mirror symmetry for singularities (one of them a joint paper of Ebeling, Gusein-Zade and Takahashi).

In [ET11] Ebeling and Takahashi defined Dolgachev triples and Gabrielov triples for invertible polynomials in three variables and established Arnold's strange duality for BHH-dual pairs.

In [ET16] they came back to [EW85] and showed how the extension of Arnold's strange duality in [EW85] follows from the BHH-duality.

In the following joint papers by Ebeling and Takahashi, and by Ebeling and Gusein-Zade, they generalized these observations in many ways, including the cases with groups G and G^T. The latest paper [EG22] by Ebeling and Gusein-Zade considers orbifold Euler characteristics, orbifold zeta functions and orbifold E-functions and proposes interesting conjectures. The two most recent papers [ET21] and [ET23] also work with matrix factorizations of size two.

The survey article [E17] discusses some homological mirror symmetry behind the mirror symmetry for invertible polynomials. The third joint paper [EP13] with David Ploog constructed in this framework objects which led to Coxeter-Dynkin diagrams of many invertible polynomials which are in the families of bimodal singularities. This paper can also be seen within the vein of homological mirror symmetry for singularities.

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