Preface

The proposal to prepare a special volume in the Journal of Singularities with contributions to the memory of Egbert Brieskorn was made by Andrew Ranicki in autumn 2017, after reading the article by *Greuel - Purkert*. The editors gladly accepted this proposal and contacted a number of students and colleagues who had worked with Brieskorn or were influenced by his work. Many of them agreed to contribute and the result is the present volume collecting the refereed papers that were submitted.

We are most grateful to the authors for their positive response and by their scientific contribution to the volume. Thanks also to the many referees for their readiness, their careful and sometimes very laborious work, and their keen judgments. We also like to thank David Massey and the Journal of Singularities for accepting our proposal for a special volume in honor of Brieskorn.

An essential feature of singularity theory is that it combines methods from different branches of mathematics, from algebraic topology over complex analysis to algebra, algebraic geometry, and Lie theory. We tried to group the papers according to related subjects. This is of course not perfect, as some papers would also fit in at least one other category, but we hope that the grouping corresponds to the main area to which each paper is ascribed.

0. The first three papers are of historical nature.
The articles by *Brieskorn* and *Hirzebruch* are reproduced reports from their talks given at the workshop “Singularitäten” in 1996 at the Mathematisches Forschungsinstitut Oberwolfach (MFO), and which never appeared elsewhere. The editors would like to thank Heidrun Brieskorn, the Hirzebruch family, and the MFO for permission to reproduce their reports in this volume.
The article by *Greuel - Purkert* describes Brieskorn's mathematical and evocative work and his life from a personal point of view.

I. Complex analytic methods lie at the heart of singularity theory. A fundamental contribution was the analytic description of the monodromy by Brieskorn. *Brasselet - Sebastiani* give a sketch of Brieskorn's fundamental manuscripta paper from 1970, explaining some central ideas in the style of that time.
Brieskorn introduced in that paper certain important concepts like the Gauß-Manin connection, a connection on a certain vector bundle, in the local situation.
Hamm - Lê look at connections in general and realize that line bundles with connection allow a much more complete theory than vector bundles with connection.
By definition connections involve differential forms. These form the subject of different papers: *Barlet* studies meromorphic differential forms with a good

pull-back property, *Dimca - Greuel* look a differential 1-forms on curves, connecting them with several geometric invariants (and offer an interesting conjecture relating the Milnor and the Tjurina number), while *Schulze - Tozzo* look at a generalization of K. Saito's free divisors, passing from divisors to complete intersections.
There is a bridge from differential forms to foliations and *Campillo - Olivares* look at the relation between foliations on a surface and their singular set.
A new central subject in Brieskorn's paper is the so-called Brieskorn lattice, the importance of which has only been realized much later. *Sabbah* looks at it in the global context for a tame function and *M. Saito* treats the uniqueness of sections of the Brieskorn module. In a long and fundamental paper *Gauss - Hertling* use the Brieskorn lattice and other invariants to determine an isolated hypersurface singularity up to right equivalence.
The Gauß-Manin connection allows also to study the eigenvalues of the monodromy. These are related to the Bernstein-Sato polynomial, too, which is studied by *Artal - Cassou-Noguès - Luengo - Melle Hernandez* in their paper.

II. There is a group of papers which deal with topology or real algebraic objects.
Classical homology theory is a basic tool, in the presence of singularities modifications of it are useful (e.g. intersection homology). *Kreck* generalizes in a different direction, comparing singular homology with bordism theory.
Traditionally singularity theory deals with complex singularities but it is natural to consider real ones, too. *Leviant - Shustin* study morsifications of these in the case of real plane curve singularities with some of their branches complex conjugate. *Oka* has discovered that several results which hold for complex polynomials hold also for "mixed" polynomials, in the present paper he focuses on the fundamental group. A classical question which refers to certain semi-algebraic objects has been taken up by *Vassiliev*, considering the following question: when does the volume of a space obtained by cutting a bounded domain with a half-space depend locally algebraically on the defining inequality of the latter?

III. Finally there are some papers which belong to the algebraic resp. algebro-geometric context, in quite different respects.
Recall that singularity theory started with isolated singularities of a holomorphic function, that is with the local case.
There are different analogues in the global case: *Damon* studies the global Milnor fibre in case of matrices (also matrices which are symmetric or skew-symmetric). *Libgober - Settepanella* look at a certain type of hyperplane arrangements.
Brieskorn was fascinated by the appearance of finite subgroups of $Sl_2(\mathbb{C})$ in singularity theory; *Ebeling* studies the MacKay correspondence for certain finite subgroups of $Sl_3(\mathbb{C})$.
Apart from singular homology the fundamental group has been from the beginning an object of study in singularity theory, especially the fundamental group of the complement of a discriminant. *Lönne* deals with a conjecture,

which was already formulated by Brieskorn in this context in 1972.
When discussing singularities one can expect more precise results by restricting to more special situations, for instance surfaces: *Némethi* looks at a class of normal surface singularities which look quite special but allow more comprehensive results, *Stevens* considers Kulikov singularities - these arise from families of curves.
A surprise is the title of the paper by *Goldman - Salman - Yomdin*: it refers to neuroscience. It turns out that questions from algebraic geometry are basic here, the paper deals with Prony systems of polynomials which are important in this context.
Varchenko treats a question from Lie theory: how to find common eigenvectors of Gaudin operators. In fact he passes from $\mathbb{C}$ to $\mathbb{F}_p$!
Deformation theory is an important branch of singularity theory; *Laudal* studies deformations of thick points - in fact non-commutative deformations.

The articles in this special volume confirm that singularity theory is nowadays a widely branched and still active subject. But it is worth while to keep common roots in mind, and Brieskorn's work plays a fundamental role here.

The three editors knew Egbert Brieskorn from the very beginning of their scientific career and profited a lot from his ideas, stimulation and encouragement. Brieskorn has been the teacher of two of us (Greuel and Hamm) and he influenced also the career of Lê, in particular by initiating the long-lasting collaboration with Hamm. It is our great pleasure to express with this special volume our gratitude for his many years of support, for his great contributions to singularity theory and his visionary leadership in the field that has influenced a generation of mathematicians.

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